

Mass Measurement in Boosted Decay Chains with Missing Energy

Jay Hubisz and Jing Shao

201 Physics Building, Syracuse University, Syracuse, NY 13244

(Dated: October 6, 2010)

We explore a novel method of mass reconstruction in events with missing transverse momentum at hadron colliders. In events with sizeable boost factors in the final steps of dual multi-stage decay chains, the missing energy particles may each be approximately collinear with a visible standard model particle, spanning a narrow “MET-cone.” We exploit this collinear approximation, when applicable, to reconstruct the masses of exotica.

Introduction — The start of the Large Hadron Collider (LHC) at CERN gives hope to the discovery of long-anticipated TeV scale new physics. In many of these scenarios, discrete symmetries are motivated by the requirement that higher dimensional operators which contribute to processes such as weak gauge boson form factors and baryon number violation must be highly suppressed. Such discrete symmetries may simultaneously be responsible for stabilizing the dark matter component of our universe. The typical collider signatures of such discrete symmetries are characterized by events with large amounts of missing transverse momentum. The undetected particles in such events, the lightest particle charged under the discrete symmetry, complicate the reconstruction of the masses of new exotica. This is particularly the case at hadron colliders, where the initial parton momenta are unknown. There has been recent substantial progress in mass measurement in such scenarios over the past few years (see [1] for a recent review, and complete citation list). Most of these methods relies on the kinematics of the events and fall into several broad categories: invariant mass endpoint methods [2–4], mass relation/polynomial methods [5–8], M_{T2} -like methods [9–11], and various combinations of them. The existence of multiple methods is crucial, providing complementary techniques for extracting information about the underlying physics model.

In this letter, we explore a conceptually new method of mass determination that is particularly useful when a decay chain terminates with the disintegration of a relatively boosted exotic particle to the lightest exotic plus a visible standard model (SM) particle.¹ Such events are characteristic of models in which pair produced color charged exotica are quite heavy, and in which there is simultaneously a small amount of phase space available for the NLSP decay. As a motivator for such scenarios, in the MSSM, the LEP II bound on the higgs mass prefers

a large top squark mass, or in general TeV scale squark masses in models with minimal flavor violation. On the other hand, the neutralinos and charginos can naturally be light, near the weak-scale and with relatively small mass splitting.

We consider production of a generic new heavy particle Q in a collider, which decays in the following way: $Q \rightarrow \cdots \chi_2 \rightarrow \cdots \chi_1 X$. Here $\chi_{1,2}$ are the LSP and NLSP respectively, while X is a SM particle. The set of dots represents a multiplicity of SM states arising from the intermediate stages of the decay chain. This rather general decay topology is shown in Fig. 1. If the mass difference between Q and χ_2 is large, $m_Q - m_{\chi_2} \gg m_{\chi_2}$, the daughter particle χ_2 will often be highly boosted.

We point out that for a given X momentum configuration, there is a kinematic boundary for the missing momentum. In the case of a boosted decay chain, the total missing momentum is constrained to lie in a narrow cone around the total X momentum, which we call the MET-cone. This observation is the central focus of this letter, and motivates the construction of a simple variable which contains information about the mass spectrum. We begin with a brief discussion of collinearity and its dependence on the mass parameters.²

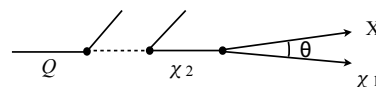


FIG. 1: A schematic picture of a boosted decay chain.

Collinearity and the MET-Cone — The kinematics of two-body χ_2 decays are straightforward. We take the χ_2 particle to have relativistic boost factor γ and velocity β in the lab frame. The angle $\theta_{\chi_2 X}$ between the visible particle X and the direction of motion of the parent χ_2 is then given by

$$\tan \theta_{\chi_2 X} = \frac{\beta_0^X}{\gamma} \left(\frac{\sin \theta_0}{\beta_0^X \cos \theta_0 + \beta} \right). \quad (1)$$

¹ From here on, we use the semantics of supersymmetry, and refer to the lightest particle charged under the discrete symmetry as the LSP, and the next lightest as the NLSP. However, our method is generic, and applies to all TeV scale physics in which the exotica carry a conserved Z_2 charge under which the SM fields are neutral.

² There is a recent study [12] which also uses boosted decays for mass measurement, but in a quite different way.

where θ_0 is angle between X and χ_2 in the rest frame of χ_2 , and β_0^X is the velocity of X in the rest frame of χ_2 . The angle takes on values in the range (for $\beta_0^X < \beta$):

$$0 \leq \tan \theta_{\chi_2 X} \leq \frac{\beta_0^X}{\gamma \beta} \frac{1}{\sqrt{1 - (\beta_0^X/\beta)^2}} \xrightarrow{\gamma \gg 1} \frac{\beta_0^X \gamma_0^X}{\gamma}. \quad (2)$$

The velocity β_0^X is a function of the masses of the three particles involved, and it characterizes the allowed phase space of the χ_2 decay. The angle $\theta_{\chi_2 \chi_1}$ can be obtained by exchanging m_X with m_{χ_1} in the above equations. A collinear configuration is achieved with a large γ , and with narrow phase space for the χ_2 decay.

The χ_2 boost factor is determined by several variables. As a simple example, we consider a heavy exotic, with mass m_Q which decays to a massless SM particle (e.g. a jet) and the NLSP, with mass m_{χ_2} . For $m_Q \sim 2$ TeV and $m_{\chi_2} \sim 200$ GeV, a boost factor of $\gamma = 5$ is achieved in the rest frame of Q . However, at a hadron collider the Q particle will be produced with some transverse as well as longitudinal momentum, providing a distribution of boost factors. In addition, in multistage decay chains, the typical boost factors will depend on the mass spectrum of particles participating in the cascade.

The boost factors of X and χ_1 in the lab frame are given by

$$\gamma_{\chi_1, X} = \gamma \gamma_0^{\chi_1, X} (1 \pm \beta \beta_0^{\chi_1, X} \cos \theta_0) \quad (3)$$

The magnitudes of the 3-momenta of X and χ_1 in the lab frame can be written as $p_{\chi_1} = \gamma_{\chi_1} \beta_{\chi_1} m_{\chi_1}$ and $p_X = \gamma_X \beta_X m_X$.

Eq.'s (2,3), define a kinematic boundary on the contribution of one χ_1 to the total \cancel{E}_T . These kinematic endpoints persist when there are two χ_1 particles in a single event. We illustrate this in Figure 2, where we display the region allowed for the total \cancel{E}_T vector in each event for given NLSP and LSP masses. We assume an event topology where all \cancel{E}_T arises from two χ_1 particles (i.e. there are no neutrinos in the event), and that each of the two decay chains terminates with the NLSP decaying to a Z -boson plus the LSP. We restrict to a particular configuration of Z momenta. The y -axis reflects the component of the \cancel{E}_T vector parallel to the total transverse Z -momentum while the x -axis displays the remaining \cancel{E}_T vector component. As expected, the total \cancel{E}_T vector is correlated with the Z -momenta, with kinematic boundaries determined by the mass spectrum of the underlying theory.

The $m_{\chi_1}^{\text{test}}$ variable — We utilize the collinear limit to inspire a test variable whose distribution yields the masses of the exotica. In the case of small mass splitting, $\Delta m = m_{\chi_2} - m_{\chi_1} - m_X$, the decay products are not significantly relativistic in the rest-frame of the parent, χ_2 . Thus in the lab frame, where the χ_2 has relativistic velocity, the boost factors of all three particles are nearly the same, and the χ_1 particles are closely aligned with

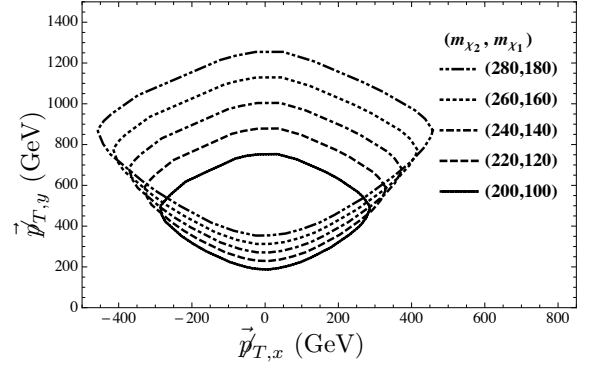


FIG. 2: In this figure we display the MET-cone boundary for different NLSP and LSP masses (keeping the mass splitting constant). The total \cancel{E}_T vector must lie within the boundary for particular choices of exotica masses. The Z bosons are in a configuration where each has boost factor 5, both lie in the transverse plane, and are separated by a 90 degree angle.

the X -momenta. With this scenario as our motivation, we define “test” missing 3-momenta as

$$\vec{p}_{\text{test}}^{a,b} \equiv \vec{p}_X^{a,b} \frac{m_{\chi_1}^{\text{test}}}{m_X}, \quad (4)$$

with $m_{\chi_1}^{\text{test}}$ defined for each event by minimization of the following quantity:

$$\Delta \cancel{E}_T^2(m_{\chi_1}^{\text{test}}) = \left| \vec{p}_{\text{test}}^{T, \text{total}} - \vec{p}_{\text{exp}}^T \right|^2. \quad (5)$$

This is an analytic procedure, as this formula is quadratic in $m_{\chi_1}^{\text{test}}$. The minimization results are given by $m_{\chi_1}^{\text{test}} = m_X \vec{p}_{T,y}/p_{X,T}^{\text{tot}}$ and $\Delta \cancel{E}_T^{\text{min}}/\cancel{E}_T = \left| \vec{p}_{T,x}/\cancel{E}_T \right|$. The variables $m_{\chi_1}^{\text{test}}$ and $\Delta \cancel{E}_T^{\text{min}}/\cancel{E}_T$ rescale respectively the y and x components of the \cancel{E}_T vector event-by-event.

As a means of quality control, we veto signal events in which $\Delta \cancel{E}_T^{\text{min}}/\cancel{E}_T > \epsilon$, for some sufficiently small ϵ . The efficiency of such a cut is itself a rough measure of the mass splitting. In Figure 3, we show rescaled MET-cones in the $m_{\chi_1}^{\text{test}}$ vs $\Delta \cancel{E}_T^{\text{min}}/\cancel{E}_T$ plane for both a scenario where the mass splitting is very small, and another where an $\mathcal{O}(10\%)$ splitting is assumed. From these figures, one observes that $m_{\chi_1}^{\text{test}}$ has two approximate endpoints in the small $\Delta \cancel{E}_T^{\text{min}}/\cancel{E}_T$ region, where the cones all intersect the $m_{\chi_1}^{\text{test}}$ axis. These endpoints are a manifestation of the MET-cone boundaries when projected onto the transverse plane, as illustrated in Figure 4.

The limit $\epsilon \rightarrow 0$ is equivalent to an alignment condition on the momenta: $\vec{p}_{\chi_1}^{T, \text{total}} \rightarrow \vec{p}_X^{T, \text{total}} m_{\chi_1}^{\text{test}}/m_X$. In this limit, one can express $m_{\chi_1}^{\text{test}}$ in terms of the measurable parameters of the event. The result can be written as an expansion in the angular separation between X and χ_1 for both sides of the decay chain, $\theta^{a,b}$, in the near-collinear case. In a configuration where both X 's are in

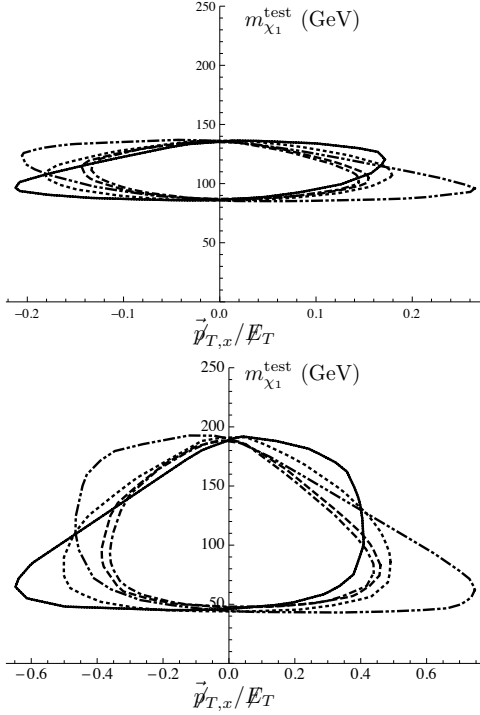


FIG. 3: In these two plots, we compare the rescaled MET-cones of two scenarios with decay $\chi_2 \rightarrow \chi_1 Z$, with identical χ_2 mass (200 GeV). In the top (bottom) plot, we take $\Delta m = m_{\chi_2} - (m_{\chi_1} + m_Z) = 1$ GeV (10 GeV). Different contours correspond to randomly chosen Z-momentum configurations. Note that $m_{\chi_1}^{\text{test}}$ has two endpoints in the small $\Delta \vec{p}_{T,x}/E_T$ region. In addition, the very small mass splitting scenario (top) predominantly occupies a much narrower range of $\Delta \vec{p}_{T,x}/E_T$.

the transverse plane, we obtain a relatively clean result:

$$m_{\chi_1}^{\text{test}} \approx m_{\chi_1} \frac{\gamma_0^{\chi_1}}{\gamma_0^X} \frac{1 + \beta \beta_0^{\chi_1} \cos \theta_0^a}{1 - \beta \beta_0^X \cos \theta_0^a} \times (1 - \cot \theta_{ab}^X \cos \phi^a \theta^a + \csc \theta_{ab}^X \cos \phi^b \theta^b), \quad (6)$$

where β and γ refer to the NLSP in the a-chain, and $\beta_0^X \approx \beta_0^{\chi_1}$ is used. Here (θ^a, ϕ^a) are the spherical coordinates of $\vec{p}_{\chi_1}^a$ in the lab frame where the z -axis is along the \vec{p}_X^a ; θ_{ab}^X is the angle between \vec{p}_X^a and \vec{p}_X^b . At zeroth order in the $\theta^{a,b}$ expansion, the endpoints of $m_{\chi_1}^{\text{test}}$ are given by:

$$m_{\pm}^{\text{test}} \approx m_{\chi_1} \frac{\gamma_0^{\chi_1}}{\gamma_0^X} \frac{1 \pm \beta_0^{\chi_1}}{1 \mp \beta_0^X}, \quad (7)$$

which are achieved when $\theta_0^a = 0$ and π respectively, i.e. χ_1 moving forward or backward along the χ_2 boost direction in its rest frame. The range of $m_{\chi_1}^{\text{test}}$ becomes smaller as the phase space for the χ_2 decay shrinks. In the relativistic limit, $\beta \sim 1$, the endpoint positions m_{\pm}^{test} approximately determine the unknown masses m_{χ_1} and m_{χ_2} .

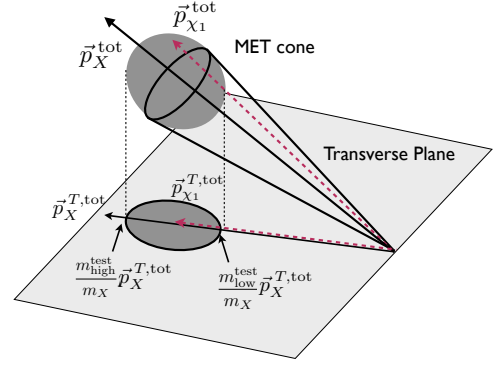


FIG. 4: Illustration of the MET cone and its relation to the $m_{\chi_1}^{\text{test}}$ variable.

Now let us discuss the non-collinear corrections to the endpoint positions from Eq. (6). First we note that θ^a depends on θ_0^a and in the near-collinear case it does not shift the endpoint positions. The dominant contribution comes from θ^b which is independent of θ_0^a and gives rise to a shift of the endpoint position $\Delta m_{\pm}^{\text{test}} \approx \pm m_{\pm}^{\text{test}} \csc \theta_{ab}^X \theta^b$. Since θ^b follows a distribution determined by the kinematics of the decay, it leads to a smearing of the distribution near these endpoints. In the near-collinear case, the variation of θ^b around the central value is small, and smearing is minimal.

In the general case where the X 's are not in the transverse plane, one needs to project the MET cone into the transverse plane and then impose the alignment condition. One can still express the result in a collinear expansion. The zeroth-order result remains the same as that in Eq. (6). However, the higher order expansion coefficients are modified by trigonometric functions expected to be of order one.

In summary, if the endpoint positions of the $m_{\chi_1}^{\text{test}}$ distribution are measured from the data, we can find solutions for the masses of the LSP and NLSP using the relation in Eq. (7).

Numerical Results — We now explore the effectiveness of the $m_{\chi_1}^{\text{test}}$ method using Monte-Carlo simulation. We consider squark pair production $\tilde{q}_L \tilde{q}_L$ followed by the decays $\tilde{q}_L \rightarrow q \tilde{\chi}_2 \rightarrow q \tilde{\chi}_1 Z$ in SUSY models. We consider the four spectra shown in Table I. For each of

Model	m_{χ_1}	m_{χ_2}	$m_{\tilde{q}_L}$	$(m_{-}^{\text{test}})^{\text{theo}}$	$(m_{+}^{\text{test}})^{\text{theo}}$
1	100	200	1000	54.6	183.2
2	100	250	1250	21.6	463.0
3	200	300	1000	117.9	339.2
4	200	350	1250	52.6	761.0

TABLE I: The relevant masses in four SUSY models and the expected endpoints $(m_{\pm}^{\text{test}})^{\text{theo}}$. Masses are given in GeV.

these models, we simulate 20k events of squark pair production and decay in pp collisions at $\sqrt{s} = 14$ TeV in MadGraph [13] using the $2 \rightarrow 6$ matrix element. Events

are selected according to the parton-level cuts in Table II. The $\cos\theta^{ab}$ cut is to ensure the coefficients in the $\theta^{a,b}$ expansion are not too large, which would obscure the endpoints. For Models 2 and 4, we have slightly loosened the selection cuts in order to get better statistics near the tail of the distribution: **a)** $\epsilon = 0.2$ for Model 2 and 4. **b)** $|\eta^{Z,\text{tot}}| < 3.0$ for Model 4.

p_T^Z	$ \eta^Z $	$ \eta^{Z,\text{tot}} $	\cancel{E}_T	ϵ	$ \cos\theta_{ab}^X $
$> 50 \text{ GeV}$	< 3.0	< 1.0	$> 200 \text{ GeV}$	0.15	< 0.5

TABLE II: Event selection cuts for $m_{\chi_1}^{\text{test}}$. Here $\eta^{Z,\text{tot}}$ is the pseudorapidity of the total Z momentum.

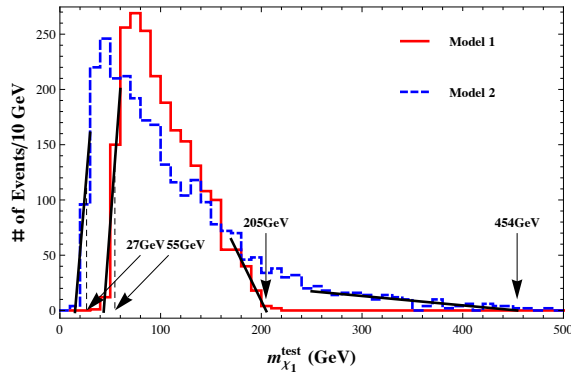


FIG. 5: The distributions of $m_{\chi_1}^{\text{test}}$ for Model 1 and 2. For Model 2, we have normalized the distribution by a factor of two.

In Fig. 5, we show the $m_{\chi_1}^{\text{test}}$ distribution for Models 1 and 2. Models 3 and 4 are very similar. For Model 1, we can see that the distribution is approximately a triangle with two endpoints at around 50 and 200 GeV. The shape of the distribution near each endpoint is slightly smeared due to deviations from collinearity. A simple way to extract these endpoints can be achieved by a linear fit and taking the x-intercept. This would typically under-estimate m_-^{test} while over-estimating m_+^{test} . In our analysis, we take the position at the half maximum near the lower edge for m_-^{test} to get a better estimation. For the upper endpoint, we use the x-intercept of a linear fit. More complicated fits and estimation are certainly possible.

The results are shown in Table III together with the statistical errors. The results are consistent with the expected zeroth-order endpoints, as shown in Table I. For Model 2, as seen in Fig. 5, the upper endpoint is much less sharp than the lower one. Estimation of its position is subject to a relatively large systematic uncertainty depending on the binning and the choice of the fitting region. Fortunately, the calculated masses are not very sensitive to the upper endpoint position. For a reasonable estimate of the m_+^{test} in the range 400–500 GeV, the calculated masses ($m_{\chi_1}^{\text{meas}}, m_{\chi_2}^{\text{meas}}$) vary only mildly from (103 GeV, 241 GeV) to (116 GeV, 264 GeV). Therefore,

Model	m_{χ_1}	m_{χ_2}	m_-^{test}	m_+^{test}	$m_{\chi_1}^{\text{meas}}$	$m_{\chi_2}^{\text{meas}}$
1	100	200	55 ± 2	205 ± 3	106 ± 2	208 ± 3
2	100	250	27 ± 2	454 ± 20	110 ± 5	253 ± 5
3	200	300	112 ± 5	342 ± 10	195 ± 5	296 ± 5
4	200	350	49 ± 2	682 ± 16	183 ± 5	329 ± 5

TABLE III: Results of the measured $m_{\chi_1}^{\text{test}}$ endpoints and $m_{\chi_{1,2}}$ for four SUSY models. Masses are given in GeV.

even in this relatively less collinear case we still obtain a good estimate of the masses. For all four models, the estimated endpoints and calculated masses are summarized in Table III. The fitted masses are all within $\sim 10\%$ of the true masses.

Summary and Outlook— In this letter, we explored a novel method of measuring the absolute mass scale of exotic particles in events with missing energy which is inspired by boosted cascade decays. This method uses the fact that in the boosted decay there is a limited variation in both the direction and magnitude of the total three-momentum of missing particles relative to the total three-momentum of the visible partners. The boundary of the allowed region, or the MET-cone, is determined by the mass parameters and the configuration of the visible particles. We constructed a variable $m_{\chi_1}^{\text{test}}$ which has endpoints in the approximate collinear case. These endpoints depend on the masses involved in the final step decay, and once observed from the data, can be used to determine these masses.

The $m_{\chi_1}^{\text{test}}$ variable works best in the collinear limit. Given the data, the evidence of collinearity in the final step decay can be seen in various ways. First, one would see a peak at 0 in the $\Delta\cancel{E}_T^{\text{min}}/\cancel{E}_T$ distribution. Second, one would see well-defined endpoints in the $m_{\chi_1}^{\text{test}}$ distribution. Once the masses of χ_1 and χ_2 are measured, one can find the masses of heavier exotics upstream of the NLSP decay using more standard techniques.

We have demonstrated our method in a setup with two symmetric decay chains with a two-step cascade decay, however it applies for longer decay chains as well. An advantage of this method is that one does not require information on all of the visible SM particles.

The $m_{\chi_1}^{\text{test}}$ variable has the advantage that it is simple to calculate on an event-by-event basis. However, it does not fully utilize the information available in the MET-cones. Developing a more effective method which takes full advantage of these kinematic boundaries is currently under investigation.

Acknowledgements — We would like to thank Yuri Gershtein, Gordon Kane, Myeonghun Park, Maxim Perelstein and Felix Yu for useful discussions. JH and JS thank Cornell University for their hospitality where part of this research was conducted. JS would also like to thank the Center for High Energy Physics at Peking University and Shanghai Jiaotong University for their hospitality where part of the research was conducted.

This work of JS is supported by the Syracuse University College of Arts and Sciences. JH is supported by the Syracuse University College of Arts and Sciences, and by the U.S. Department of Energy under grant DE-FG02-85ER40237.

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